NAMBU-JONA-LASINIO MODEL IN CURVED SPACETIME WITH MAGNETIC FIELD

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Abstract

We discuss the phase structure of the NJL model in curved space-time with magnetic field using $1/N$-expansion and linear curvature approximation. The effective potential for composite fields $\bar{\psi}\psi$ is calculated using the proper-time cut-off in the following cases: a) at non-zero curvature, b) at non-zero curvature and non-zero magnetic field, and c) at non-zero curvature and non-zero covariantly constant gauge field. Chiral symmetry breaking is studied numerically. We show that the gravitational field may compensate the effect of the magnetic field what leads to restoration of chiral symmetry.

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1 Introduction

The role of the magnetic fields in the models of the inflationary Universe has been recently discussed in refs. [1,2]. The realization of the fact that strong primordial magnetic fields should be considered on equal footing with the strong curvature in the early Universe may lead to some new effects in different cosmological applications. For example, the combined effect of the gravitational field and electromagnetic field can produce a significant increase in the number of created particles in the early Universe [3]. Hence, there appears the motivation to study different field theories in the combination of external gravitational and electromagnetic fields.

In the present letter we investigate the Nambu-Jona-Lasinio (NJL) model in curved spacetime with magnetic field. Such a model has been often considered in particle physics, as a reliable scenario for studying such basic phenomena as chiral symmetry breaking and the formation of composite bound states. The phase structure of the NJL model in curved spacetime is known to be quite rich [4]. The possibility of curvature induced chiral phase transitions may be realized there. From another side, the magnetic field usually supports the chiral symmetry breaking. Hence, it is interesting to study the behaviour of the chiral symmetry under the action of combined magnetic and gravitational field. Note that such a study may be relevant to particle physics due to the fact that the Standard Model may be reformulated as (gauged) NJL model.

We start in Section 2 from the description of the NJL model in curved spacetime, and we calculate the effective potential for composite fields in 1/N expansion and in linear curvature aproximation. Working in proper-time cut-off regularization, the possibility of chiral symmetry breaking induced by curvature is carefully discussed. Section 3 is devoted to the situation when the external magnetic field is also present. The effective potential is again evaluated, the corresponding gap equation is obtained and the chiral symmetry breaking under the action of external gravitational and magnetic fields is numerically investigated for some values of the coupling constant. In Section 4 we show how to find the effective potential for the NJL model in curved spacetime with covariantly constant external gauge field.

2 NJL model in curved spacetime: proper-time cut-off

Let us start from the action for the Nambu-Jona-Lasinio model in curved space-time:

$$S = \int d^4x \sqrt{-g} \left(\bar{\psi} i \gamma^\mu(x) \nabla_\mu \psi + \frac{\lambda}{2N} \left((\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right) \right) \qquad (1),$$

where the spinor ψ has N components and $\gamma^\mu(x)$ are the curved spacetime Dirac matrices.

The standard way to study this model is to work in the scheme of the 1/N expansion. We will limit ourselves to the leading order in the 1/N expansion. Introducing the auxiliary fields $\sigma(x)$ [Band $\pi(x)$ one can rewrite the action (1) in the equivalent form

$$S = \int d^4x \sqrt{-g} \left(\bar{\psi} i \gamma^\mu(x) \nabla_\mu \psi - \frac{N}{2\lambda} \left(\sigma^2 + \pi^2 \right) - \bar{\psi}(\sigma + i \gamma_5 \pi)\psi \right). \qquad (2)$$

It is known that even in flat space the global abelian chiral symmetry of above model is broken spontaneously when the coupling constant is supercritical one. In curved spacetime the situation is getting more rich [4]. There appears the possibility of curvature induced phase transitions between symmetric and non symmetric phases. In other words, symmetry may be broken even below the critical coupling constant (depending on the curvature).

In this section we will discuss the symmetry breaking in the NJL model working in the linear curvature aproximation (for a general review of the effective action in linear curvature aproximation, see [5]). This model has been already studied in the linear curvature approximation (see first paper in [4]), using local momentum representation with ultraviolet cut-off in momentum integrals.

We will work in proper-time representation [6] and will use proper-time cut-off. This method will be convenient below for generalization to the situation when in addition the external magnetic field is present. Integrating over fermionic fields in the generating functional for the theory (2) we will get the semiclassical effective action

$$S_{eff} = - \int d^4x \sqrt{-g} \left(\frac{1}{2\lambda} (\sigma^2 + \pi^2) - i \ln \det \left[i\gamma^\mu(x)\nabla_\mu - (\sigma + i\gamma_5\pi) \right] \right); \quad (3)$$

note that N has been factored out.

Let us now discuss the effective potential, where as usually due to dependence of V from the invariant $\sigma^2 + \pi^2$ it is enough to put $\pi = 0$. Using the same technique as in the first reference in [4] we can show that

$$V(\sigma) = \frac{\sigma^2}{2\lambda} + i \ln \det[i\gamma^\mu(x)\nabla_\mu - \sigma]. \quad (4)$$

Then, working in terms of the derivative of V with respect to σ and using linear curvature expansion for the propagator we get (see first paper in [4])

$$V'(\sigma) = \frac{\sigma}{\lambda} - iTr \int \frac{d^4k}{(2\pi)^4} \left[\frac{\hat{k}+\sigma}{k^2-\sigma^2} - \frac{R}{12} \frac{\hat{k}+\sigma}{(k^2-\sigma^2)^2} \right. $$
$$\left. + \frac{2}{3} R_{\mu\nu} k^\mu k^\nu \frac{\hat{k}+\sigma}{(k^2-\sigma^2)^3} - \frac{1}{2} \gamma^a \sigma^{cd} R_{cda\mu} \frac{k^\mu}{(k^2-\sigma^2)^2} \right] \quad (5)$$

where $\hat{k} = \gamma^\mu k_\mu$. After the Wick rotation and calculation of trace we obtain

$$V'(\sigma) = \frac{\sigma}{\lambda} + \frac{\sigma}{(2\pi)^2} \int_0^\infty k^3 dk \left[-\frac{1}{k^2+\sigma^2} - \frac{R}{12(k^2+\sigma^2)^2} + \frac{R}{6} \frac{k^2}{(k^2+\sigma^2)^3} \right], \quad (6)$$

The expression (6) may be rewritten in the following way

$$V'(\sigma) = \frac{\sigma}{\lambda} + \frac{\sigma}{(2\pi)^2} \int_{1/\Lambda^2}^\infty ds \left(-\frac{1}{s^2} + \frac{R}{12s} \right) e^{-s\sigma^2}. \quad (7)$$

where the representation

$$\frac{1}{(k^2+\sigma^2)^\nu} = \frac{1}{\Gamma(\nu)} \int_0^\infty ds s^{\nu-1} \exp(-s(k^2+\sigma^2)) \quad (8)$$

has been used, integration over k has been done and the ultra-violet proper-time cut-off is introduced directly into the integral.

Integrating over σ we get

$$V(\sigma) = \frac{\sigma^2}{2\lambda} + \frac{1}{8\pi^2} \int_{1/\Lambda^2}^{\infty} ds \exp(-s\sigma^2) \left[\frac{1}{s^3} - \frac{R}{12s^2} \right]$$

$$= \frac{\sigma^2}{2\lambda} + \frac{1}{(4\pi)^2} \left[(\Lambda^4 - \Lambda^2\sigma^2) \exp(-\frac{\sigma^2}{\Lambda^2}) \right.$$

$$\left. + \sigma^4 Ei(\frac{\sigma^2}{\Lambda^2}) - \frac{R}{6} \left(\Lambda^2 \exp(-\frac{\sigma^2}{\Lambda^2}) - \sigma^2 Ei(\frac{\sigma^2}{\Lambda^2}) \right) \right], \tag{9}$$

where $Ei(x) = \int_x^{\infty} \frac{\exp(-t)}{t} dt = -\gamma - \ln x - \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{nn!}$ and γ is the Euler constant.

Expanding Eq. (9) and keeping only terms which are not zero at $\Lambda \to \infty$ we get

$$V(\sigma) = \frac{\sigma^2}{\Lambda^2} - \frac{1}{(4\pi)^2} \left[2\Lambda^2\sigma^2 + \sigma^4 \left(\ln \frac{\sigma^2}{\Lambda^2} + \gamma - \frac{3}{2} \right) \right.$$

$$\left. + \frac{R\sigma^2}{6} \left(\ln \frac{\sigma^2}{\Lambda^2} + \gamma - 1 \right) \right] + 0(\frac{1}{\Lambda^2}) \tag{10}$$

Thus, we have got the effective potential with proper-time cut-off.

Using Eq.(10) the gap equation is found as follows

$$\frac{4\pi^2}{\lambda\Lambda^2} - 1 = \frac{\sigma^2}{\Lambda^2} \left(\ln \frac{\sigma^2}{\Lambda^2} + \gamma - 1 \right) + \frac{R}{12\Lambda^2} \left(\ln \frac{\sigma^2}{\Lambda^2} + \gamma \right). \tag{11}$$

In flat space-time it is well known that the chiral symmetry is broken only for the coupling constant above some λ_0

$$\lambda \geq \lambda_0 = \frac{4\pi^2}{\Lambda^2}. \tag{12}$$

In curved spacetime, depending on the coupling constant and on the curvature, the situation is more interesting [4]. Particulary, for some fixed λ one can find the possibility of curvature induced first order phase transition (see [4]). In Fig. 1 we have plotted the effective potential $V(\sigma, 0)$ for different values of R (given in units of Λ^2 and for the coupling $\lambda = 1.25\lambda_0$).

The solution of the gap equation (11) gives us the vacuum expectation value of the composite field $\bar{\psi}\psi$ and is equal to the dynamical mass of the fermion. In Fig. 2 we have plotted the dynamical mass of the fermion field as a function of the curvature R for fixed $\lambda = 1.25\lambda_0$.

3 NJL model in curved spacetime with magnetic field

Let us discuss now the situation when the NJL model is considered under the action of external gravitational and magnetic fields. The magnetic field may be treated exactly in the proper-time method [6] (for a review of the proper-time method in external electromagnetic fields see [7]). It is quite known that the magnetic field increases strongly the possibility of the dynamical symmetry breaking (see, for example [8]), if compared with the situation without it. The external gravitational field will be taken into account in linear curvature aproximation discussed in the previous section (this aproximation is perfectly enough to take into account the gravitational effects even in GUT epoch). Moreover, in linear curvature aproximation we discuss only curvature terms which explicitly do not depend on the less important linear curvature-magnetic contributions.

With all these remarks, repeating basically the steps of section 2 (the covariant derivative is now $\tilde{\nabla}_\mu = \nabla_\mu - ieA_\mu$, $A_\mu = -Bx_2\delta_{\mu 1}$) one can get the following effective potential

$$V(\sigma) = \frac{\sigma^2}{2\lambda} + iTr\ln\left[i\gamma^\mu(x)\tilde{\nabla}_\mu - \sigma\right]$$

$$= \frac{\sigma^2}{2\lambda} + \frac{1}{8\pi^2}\int_{1/\Lambda^2}^\infty \frac{ds}{s^2}\exp(-s\sigma^2)\left[|eB|\coth(s|eB|) + \left(-\frac{R}{12} + 0(R^2)\right)\right]. \quad (13)$$

In the absence of the gravitational field ($R = 0$), the effective potential corresponds to flat space situation where the magnetic field is treated exactly [6]. In the absence of the magnatic field ($B = 0$) we are back to the potential (9). In addition, in the linear curvature terms the effect of the magnetic field is not taken into account (it will be shown in another place that such terms are not relevant compared with the terms written in (13)).

Making the calculation of the integrals in Eq. (13) up to $0(1/\Lambda^2)$, and taking the derivative with respect to σ one gets the gap equation

$$\frac{\partial V}{\partial \sigma} = 0 \quad (14)$$

as follows

$$\frac{4\pi^2}{\lambda\Lambda^2} - 1 = -\frac{\sigma^2}{\Lambda^2} \ln \frac{(\Lambda|eB|^{-1/2})^2}{2} + \frac{|eB|}{\Lambda^2} \ln \frac{(\sigma|eB|^{-1/2})^2}{4\pi} + \gamma\frac{\sigma^2}{\Lambda^2}$$

$$+ \quad 2\frac{|eB|}{\Lambda^2} \ln \Gamma\left(\frac{\sigma^2|eB|^{-1}}{2}\right) - \frac{R}{12\Lambda^2}\left(\ln \frac{\Lambda^2}{\sigma^2} - \gamma\right) + 0(\frac{1}{\Lambda}). \quad (15)$$

Using this gap equation one can study the dynamical symmetry breaking in different cases. In some cases it can be given analytically, for example, for values of the coupling constant λ much below the critical value, i.e.,

$$\lambda \ll \frac{4\pi^2}{\Lambda^2}. \quad (16)$$

One can find (supposing that the second term in the r.h.s. of (15) is the leading one)

$$\frac{4\pi^2}{\lambda\Lambda^2} - 1 \approx \frac{|eB|}{\Lambda^2} \ln \frac{(\sigma|eB|^{-1/2})^2}{4\pi} - \frac{R}{12\Lambda^2} \ln \frac{\Lambda^2}{\sigma^2} \quad (17)$$

and finally, the dynamically generated fermionic mass is given by

$$\sigma^2 \approx \left[\left[\frac{|eB|^{-1}}{4\pi}\right]^{-|eB|/\Lambda^2} (\frac{1}{\Lambda^2})^{-R/(12\Lambda^2)} \exp\left(\frac{4\pi^2}{\lambda\Lambda^2} - 1\right)\right]^{1/(|eB|/\Lambda^2 + R/(12\Lambda^2))}$$

$$(18)$$

In the absence of the magnetic field it gives the analytic expression for the dynamically generated fermionic mass due to the curvature

$$\sigma^2 \approx \Lambda^2 \exp\left(\frac{12\Lambda^2}{R}\left[\frac{4\pi^2}{\lambda\Lambda^2} - 1\right]\right). \quad (19)$$

One can see that positive curvature tends to make the first term in (17) less, i.e. it acts against the dynamical symmetry breaking. At the same time, the negative curvature always favors the dynamical chiral symmetry breaking in accordance with the explicit calculations in external gravitational field [4] and with general considerations of ref. [9].

One can give the behaviour of the effective potential as a function of curvature and magnetic field for fixed four-fermion coupling constant. Fig. 3 shows the breakdown of the symmetry by the magnetic field ($|eB|/\Lambda^2 = 0.1$)

and the restoration of the symmetry by the gravitational field at $(R/\Lambda^2 = 0.9)$, where the coupling constant λ takes the value $\lambda = 1.25\lambda_0$, where λ_0 is given in Eq. (12). In agreement with Eq. (17), negative values of curvature favor symmetry breaking. Fig. 4 shows the same effect for the coupling $\lambda = 0.5\lambda_0$. It is seen from Figs. 3 and 4 that a second order phase transition takes place as R changes. In Fig. 5 we have plotted the solution of the gap equation (15) as a function of R for two different values of the coupling λ with the same magnetic field.

4 NJL model in curved spacetime with covariantly constant gauge field

In this section we will discuss the NJL model within covariantly constant gauge field and in a weakly curved spacetime. We consider massless NJL model of ref. [10] with $SU(2)_R \times SU(2)_L$ chiral symmetry. The model is described by the Lagrangian

$$L = \bar{q}i\gamma^\mu \bar{\nabla}_\mu q + g\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2\right],\qquad(20)$$

where the external $SU(3)$ gauge field is considered, q is the spinor doublet of $SU(3)$ (with components $q^a, a = 1, ...8$). Working with covariantly constant $SU(3)$ gauge field expressed in terms of invariants H and E,

$$H\,\text{resp.}\,E = \left\{\left[(\frac{1}{4}G^{a\,2}_{\mu\nu})^2 + (\frac{1}{4}G^a_{\mu\nu}\tilde{G}^a_{\,mu\nu})^2\right]^{1/2} \pm \frac{1}{4}G^{a\,2}_{\mu\nu}\right\}^{1/2},\qquad(21)$$

and again expressing the effective potential in terms of the auxiliary fields (for details in flat space, see [10]) one can get

$$V = \frac{\sigma^2 + \pi^2}{4g} + \frac{N_f}{8\pi^2}tr_c PV \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^3} e^{-s(\sigma^2+\pi^2)}(g^2 EHs^2)\cot(gEs)\coth(gHs)$$

$$-i\frac{N_f}{8\pi^2}tr_c \sum_{n=1}^{\infty} \frac{g^2 HE}{n}\coth(HE^{-1}n\pi)\exp\left[-\frac{n\pi(\sigma^2+\pi^2)}{gE}\right]$$

$$-\frac{N_f}{8\pi^2}\frac{R}{12}tr_c \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^2}\exp(-s(\sigma^2+\pi^2)),\qquad(22)$$

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where N_f corresponds to the flavor symmetry, tr_c means trace over color space and PV means the principal value. The same cut-off regularization is applied. The imaginary part of the effective potential describes the particle creation (for a review, see [7]).

Using the real part of the above effective potential one again can study numerically the dynamical symmetry breaking under the action of covariantly constant gauge field and weak gravitational field. The qualitative behaviour of the effective potential in chromomagnetic field taking into account the curvature effects is similar to the behaviour in the previous section. We do not discuss this model in detail because the vacuum is not stable due to the presence of the imaginary part in an effective potential.

In summary, in this paper we studied the NJL model in curved spacetime with magnetic field. Working in linear curvature expansion and in the proper-time cut-off, the effective potential for the composite field $\bar{\psi}\psi$ has been calculated. The phase structure of the theory has been studied numerically first for the case without magnetic field. In a situation, when the magnetic field is not zero the phase structure has been studied under the combined action of magnetic field (which gives stronger effect to dynamical symmetry breaking) and of gravitational field. Finally, we have shown how one can extend the results of the present study to the background of curved spacetime with covariantly constant gauge fields.

The approach described in this paper may be extended to the situation when both gravitational and magnetic fields are treated exactly (with some technical details being more complicated), or to other background on gauge fields (say, external electromagnetic field).

The results of the above study maybe useful for cosmological applications (in particulary, the early Universe) in inflationary models based on the composite fields potential of sort (10), (13). Note also that it would be of interest to discuss more realistic gauged NJL model in a similar way.

References

[1] M.S. Turner and L.M. Widrow, Phys. Rev. D37 (1987), 2743

[2] M. Gasperini, M. Giovannini and G. Veneziano, Phys. Rev. Lett. (1995)

[3] I.L. Buchbinder and S.D. Odintsov, Izw. VUZov Fizika (Sov. Phys. J.) N. 5 (1982), 12.

[4] T. Inagaki, T. Muta and S.D. Odintsov, Mod. Phys. Lett. A8 (1993), 2117; E. Elizalde, S. Leseduarte and S.D. Odintsov, Phys. Rev. D49 (1994), 5551; E. Elizalde and S.D. Odintsov, Phys. Rev. D51 (1995), 5990; T. Inagaki, S. Mukaigawa and T. Muta, Phys. Rev. D52 (1995), 4267; E. Elizalde, S. Leseduarte, S.D. Odintsov and Yu. I. Shilnov, Phys. Rev. D (1996); T. Inagaki, Preprint HUPD-9522, 1995.

[5] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, Effective Action in Quantum Gravity, IOP Publishing, Bristol and Philadelphia, 1992.

[6] J. Schwinger, Phys. Rev. 82 (1951), 664.

[7] E.S. Fradkin, D.M. Gitman and S.M. Shvartsman, Quantum Electrodynamics with Unstable Vacuum, Springer-Verlag, Berlin, 1991; D.M. Gitman and S.P. Gavrilov, preprint IFUSP/P-1185, 1995.

[8] I. Krive and S. Naftulin, Phys. Rev. D46, (1992), 2337; S.P. Klebansky, Rev. Mod. Phys. 64 (1992), 1.

[9] C. Callan and F. Wilczek, Nucl. Phys. B340 (1990), 366

[10] H. Suganuma and T. Tatsumi, Progr. Theor. Phys. 90 (1993), 379.

Figure 1: The behaviour of the effective potential $v = V/\Lambda^4$ as a function of x $(x = \sigma/\Lambda)$ for different values of r $(r = R/\Lambda^2)$, for fixing $\lambda = 1.25\lambda_0$

Figure 2: The solution of the gap equation (11) $(x = \sigma/\Lambda$ is shown as a function of the curvature r $(r = R/\Lambda^2)$ for $\lambda = 1.25\lambda_0$

Figure 3: The behaviour of the effective potential $v = V/\Lambda^4$ for different values of r $(r = R/\Lambda^2)$, for fixing $\lambda = 1.25\lambda_0$ and magnetic field $|eB|/\Lambda^2 = 0.1$

Figure 4: The behaviour of the effective potential $v = V/\Lambda^4$ for different values of r $(r = R/\Lambda^2)$, for fixing $\lambda = 0.5\lambda_0$ and magnetic field $|eB|/\Lambda^2 = 0.2$

Figure 5: The solution $x = \sigma/\Lambda$ of the gap equation (15) as a function of the curvature r $(r = R/\Lambda^2)$ for $\lambda = 1.25\lambda_0$ and $\lambda = 0.5\lambda_0$
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